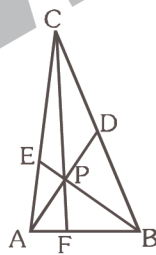
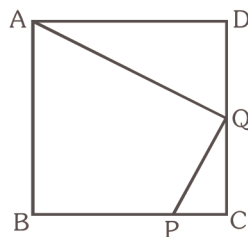


16. Given that the lengths of three sides, a, b, c of a triangle are positive integers, where $a < b < c$. Find the number of the triangles with $b = 2$.
17. In a right-angled triangle, if the length of a leg is 21, and the lengths of the other two sides are also positive integers, find the minimum value of its possible perimeter.
18. In right-angled $\triangle ABC$, $\angle C = 90^\circ$, E is on BC such that $AC = BE$. D is on AB such that $DE \perp BC$. Given that $DE + BC = 1$, $BD = \frac{1}{2}$, find $\angle B$ in degrees.
19. In $\triangle ABC$, $AC = BC$, $\angle C = 20^\circ$, M is on the side AC and N is on the side BC , such that $\angle BAN = 50^\circ$, $\angle ABM = 60^\circ$. Find $\angle NMB$ in degrees.
20. In the $\triangle ABC$, BE is the angle bisector of the $\angle ABC$, AD is the median on the side BC , and AD intersects BE at O perpendicularly. Given $BE = AD = 4$, find the lengths of three sides of $\triangle ABC$.
21. In $\triangle ABC$, D is the midpoint of BC , E is on AC such that $AC = 3EC$. BE and AD intersect at G . Find $AG : GD$.
22. When extending the sides AB, BC, CA of $\triangle ABC$ to B', C', A' respectively, such that $AB' = 2AB$, $CC' = 2BC$, $AA' = 3CA$. If area of $\triangle ABC$ is 1, find the area of $\triangle A'B'C'$.
23. Given that the point P is outside the equilateral triangle ABC but inside the region of $\angle ABC$. If the distances from P to BC, CA, AB are h_1, h_2 and h_3 respectively, and $h_1 - h_2 + h_3 = 6$, find the area of $\triangle ABC$.
24. Given that G is the centroid of $\triangle ABC$, $GA = 2\sqrt{3}$, $GB = 2\sqrt{2}$, $GC = 2$. Find the area of $\triangle ABC$.
25. Point P is inside $\triangle ABC$. Line segments APD, BPE , and CPF are drawn with D on BC , E on CA , and F on AB (see the figure below). Given that $AP = 6$, $BP = 9$, $PD = 6$, $PE = 3$, and $CF = 20$, find the area of $\triangle ABC$.

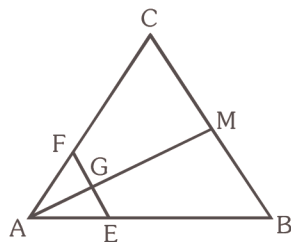


26. In a square $ABCD$, let P be a point on the side BC such that $BP = 3PC$ and Q be the mid-point of CD . If the area of the triangle PCQ is 5, what is the area of triangle QDA ?



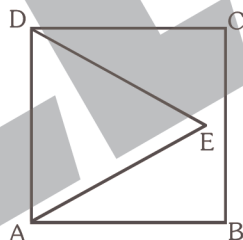
27. A triangle $\triangle ABC$ is inscribed in a circle of radius 4 cm. Suppose that $\angle A = 60^\circ$, $AC - AB = 4$ cm, and the area of $\triangle ABC$ is $x \text{ cm}^2$. Find the value of $\left(\frac{x}{2}\right)^2$.

28. In $\triangle ABC$, $AB : AC = 4 : 3$ and M is the midpoint of BC . E is a point on AB and F is a point on AC such that $AE : AF = 2 : 1$. It is also given that EF and AM intersect at G with $GF = 36$ cm and $GE = x$ cm. Find the value of x .

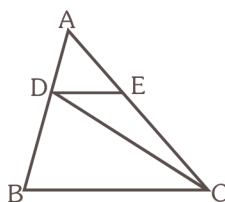


29. Nondegenerate $\triangle ABC$ has integer side lengths, BD is an angle bisector, $AD = 3$, and $DC = 8$. What is the smallest possible value of the perimeter?
30. It is known that the 3 sides of a triangle are consecutive positive integers and the largest angle is twice the smallest angle. Find the perimeter of this triangle.
31. A triangle $\triangle ABC$ is inscribed in a circle of radius 1, with $\angle BAC = 60^\circ$. Altitudes AD and BE of $\triangle ABC$ intersect at H . Find the smallest possible value of the length of the segment AH .
32. In triangle ABC , $AB = 28$, $BC = 21$ and $CA = 14$. Points D and E are on AB with $AD = 7$ and $\angle ACD = \angle BCE$. Find the length of BE .
33. Four points in the order A, B, C, D lie on a circle with the extension of AB meeting the extension of DC at E and the extension of AD meeting the extension of BC at F . Let EP and FQ be tangents to this circle with points of tangency P and Q respectively. Suppose $EP = 60$ and $FQ = 63$. Determine the length of EF .
34. The diagram shows an equilateral triangle ADE inside a square $ABCD$. What is the value of

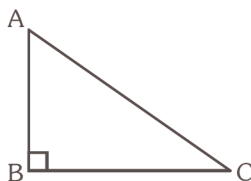
$$\left(\frac{\text{area of } \triangle ADE}{\text{area of } \triangle DEC} \right)^2$$



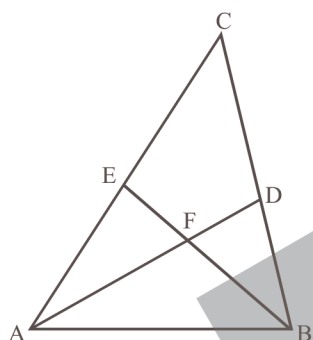
35. In this question, $S_{\triangle XYZ}$ denotes the area of $\triangle XYZ$. In the following figure, if $DE \parallel BC$, $S_{\triangle ADE} = 1$ and $S_{\triangle ADC} = 4$, find $S_{\triangle DBC}$.



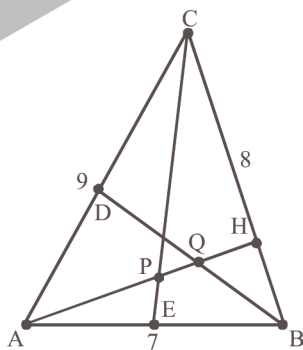
36. $\triangle ABC$ is a right-angled triangle with $\angle ABC = 90^\circ$. A circle C_1 is drawn with AB as diameter, and another circle C_2 is drawn with BC as diameter. The circles C_1 and C_2 meet at the points B and P . If $AB = 5$ cm, $BC = 12$ cm and $BP = x$ cm, find the value of $\frac{120}{x}$.



59. In $\triangle PAT$, $\angle P = 36^\circ$, $\angle A = 56^\circ$, and $PA = 10$. Points U and G lie on sides \overline{TP} and \overline{TA} , respectively, so that $PU = AG = 1$. Let M and N be the midpoints of segments \overline{PA} and \overline{UG} , respectively. What is the degree measure of the acute angle formed by lines MN and PA ?
60. Let ABC be an equilateral triangle. Extend side \overline{AB} beyond B to a point B' so that $BB' = 3AB$. Similarly, extend side \overline{BC} beyond C to a point C' so that $CC' = 3BC$, and extend side \overline{CA} beyond A to a point A' so that $AA' = 3CA$. The ratio of the area of $\triangle A'B'C'$ to the area of $\triangle ABC$ is $Q : 1$. What is Q ?
61. In $\triangle ABC$, $AB = 6$, $BC = 7$, and $CA = 8$. Point D lies on \overline{BC} , and \overline{AD} bisects $\angle BAC$. Point E lies on \overline{AC} , and \overline{BE} bisects $\angle ABC$. The bisectors intersect at F . The ratio of $AF : FD$ is $m : n$. What is $m^2 + n^2$?

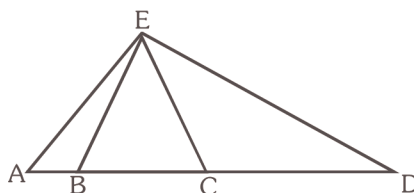


62. In $\triangle ABC$ shown in the figure, $AB = 7$, $BC = 8$, $CA = 9$, and \overline{AH} is an altitude. Points D and E lie on sides \overline{AC} and \overline{AB} , respectively, so that \overline{BD} and \overline{CE} are angle bisectors, intersecting \overline{AH} at Q and P , respectively. If $PQ = \frac{a}{b}\sqrt{c}$, where a, b are co-prime, then what is the value of $(a + b + c^2)$?

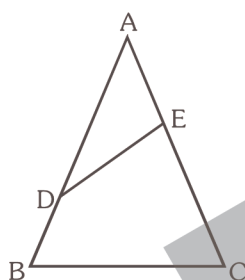


63. There are f noncongruent integer-sided triangles with positive area and perimeter less than 15, which are neither equilateral, isosceles, nor right triangles. What is f ?
64. In $\triangle BAC$, $\angle BAC = 40^\circ$, $AB = 10$, and $AC = 6$. Points D and E lie on \overline{AB} and \overline{AC} respectively. What is the minimum possible value of $BE + DE + CD$?
65. In $\triangle ABC$, $AB = AC = 28$ and $BC = 20$. Points D, E and F are on sides \overline{AB} , \overline{BC} , and \overline{AC} , respectively, such that \overline{DE} and \overline{EF} are parallel to \overline{AC} and \overline{AB} , respectively. What is the perimeter of parallelogram $ADEF$?

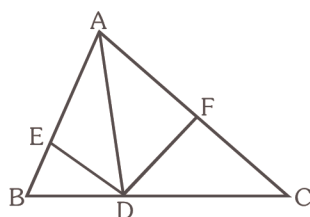
73. In triangle ABC, $AB = (b^2 - 1)$ cm, $BC = a^2$ cm and $AC = 2a$ cm, where a and b are positive integers greater than 1. Find the value of $a - b$.
74. In the figure below, ADE is a triangle with $\angle AED = 120^\circ$, and B and C are points on side AD such that BCE is an equilateral triangle. If $AB = 4$ cm, $CD = 16$ cm and $BC = x$ cm, find the value of x .



75. The figure below shows a triangle ABC where $AB = AC$. D and E are points on sides AB and AC, respectively, such that $AB = 4DB$ and $AC = 4AE$. If the area of the quadrilateral BCED is 52 cm^2 and the area of the triangle ADE is $x \text{ cm}^2$, find x .



76. In the triangle ABC, $AC = 2BC$, $\angle C = 90^\circ$ and D is the foot of the altitude from C onto AB. A circle with diameter AD intersects the segment AC at E. Find $AE : EC$.
77. In the triangle ABC, $AB = 8$, $BC = 7$ and $CA = 6$. Let E be the point on BC such that $\angle BAE = 3\angle EAC$. Find $\frac{4AE^2}{5}$.
78. In the triangle ABC, the bisectors of $\angle A$ and $\angle B$ meet at the incentre I, the extension of AI meets the circumcircle of triangle ABC at D. Let P be the foot of the perpendicular from B onto AD, and Q a point on the extension of AD such that $ID = DQ$. Determine the value of $(BQ \times IB)/(BP \times ID)$.
79. Three sides OAB, OAC and OBC of a tetrahedron OABC are right-angled triangles, i.e. $\angle AOB = \angle AOC = \angle BOC = 90^\circ$. Given that $OA = 7$, $OB = 2$ and $OC = 6$, find the value of $[(\text{Area of } \triangle OAB)^2 + (\text{Area of } \triangle OAC)^2 + (\text{Area of } \triangle OBC)^2 + (\text{Area of } \triangle ABC)^2 + 37]^{1/2}$.
80. In a triangle ABC, the length of the altitudes AD and BE are 4 and 12 respectively. Find the largest possible integer value for the length of the third altitude CF.
81. In the figure below, ABC is a triangle and D is a point on side BC. Point E is on side AB such that DE is the angle bisector of $\angle ADB$, and point F is on side AC such that DF is the angle bisector of $\angle ADC$. Find the value of $\frac{AE}{EB} \cdot \frac{BD}{DC} \cdot \frac{CF}{FA}$.



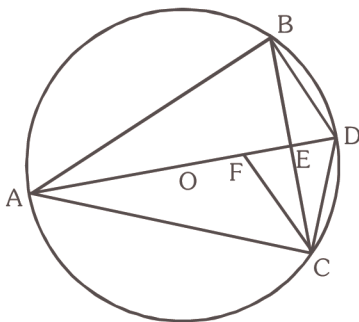
- 127.** Let ABC be an equilateral triangle. Extend side \overline{AB} beyond B to a point B' so that $BB' = 3AB$. Similarly, extend side \overline{BC} beyond C to a point C' so that $CC' = 3BC$, and extend side \overline{CA} beyond A to a point A' so that $AA' = 3CA$. If the ratio of the area of $\triangle A'B'C'$ to the area of $\triangle ABC$ is $x : 1$. Find the value of x .
- 128.** A thin piece of wood of uniform density in the shape of an equilateral triangle with side length 3 inches weighs 12 gm. A second piece of the same type of wood, with the same thickness, also in the shape of an equilateral triangle, has side length of 5 inches. If W is the weight, in gm, of the second piece. Then find $[W]$, where $[x]$ denotes the greatest integer less than or equal to x .
- 129.** The isosceles right triangle ABC has right angle at C and area 12.5. The rays trisecting $\angle ACB$ intersect AB at D and E . If the area of $\triangle CDE$ can be written as $\frac{a(2-\sqrt{b})}{2}$. Find the product of a and b .

ALLEN

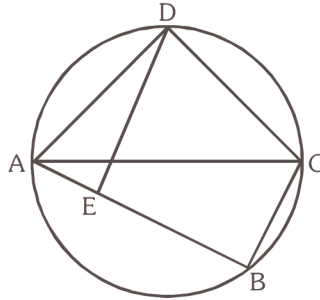
Important Notes

ALLEN

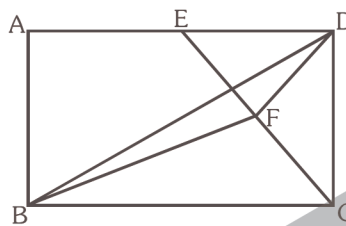
83. A circle with center O has area 156π . Triangle ABC is equilateral, \overline{BC} is a chord on the circle, $OA = 4\sqrt{3}$, and point O is outside $\triangle ABC$. What is the side length of $\triangle ABC$?
84. In the figure below, ABC is an isosceles triangle inscribed in a circle with centre O and diameter AD , with $AB = AC$. AD intersects BC at E , and F is the midpoint of OE . Given that BD is parallel to FC and $BC = 2\sqrt{5}$ cm, find square of the length CD (in cm).



43. Let ABCD be a quadrilateral inscribed in a circle with diameter AC, and let E be the foot of perpendicular from D onto AB, as shown in the figure below. If $AD = DC$ and the area of quadrilateral ABCD is 24 cm^2 , find the square of length of DE (in cm).



44. In the figure below, ABCD is a rectangle, E is the midpoint of AD and F is the midpoint of CE. If the area of triangle BDF is 12 cm^2 , find the area of rectangle ABCD in cm^2 .

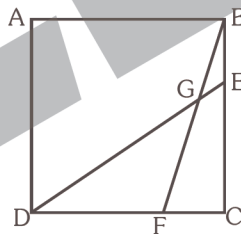


45. Given that ABCD is a square. Points E and F lie on the side BC and CD respectively, such that $BE = CF = \frac{1}{3} AB$.

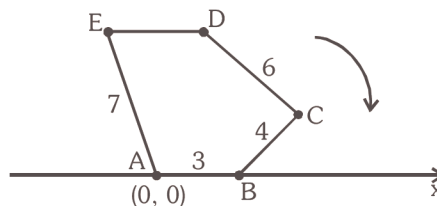
G is the intersection of BF and DE. If

$$\frac{\text{Area of ABGD}}{\text{Area of ABCD}} = \frac{m}{n}$$

is in its lowest term, find the value of $m + n$.

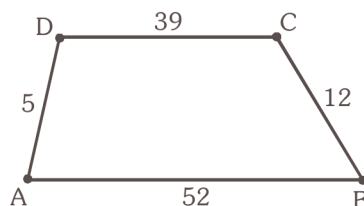


46. Rectangle ABCD has $AB = 4$ and $BC = 3$. Segment EF is constructed through B so that EF is perpendicular to DB, and A and C lie on DE and DF, respectively. What is EF to the nearest integer?
47. As shown below, convex pentagon ABCDE has sides $AB = 3$, $BC = 4$, $CD = 6$, $DE = 3$, and $EA = 7$. The pentagon is originally positioned in the plane with vertex A at the origin and vertex B on the positive x-axis. The pentagon is then rolled clockwise to the right along the x-axis. What is the length side which will touch the point $x = 2009$ on the x-axis?

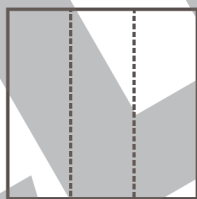


48. Rectangle ABCD has $AB = 8$ and $BC = 6$. Point M is the midpoint of diagonal \overline{AC} , and E is on \overline{AB} with $\overline{ME} \perp \overline{AC}$. What is the area of $\triangle AME$ to the nearest integer?

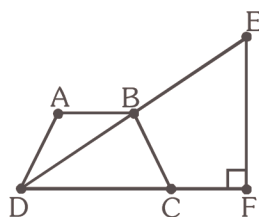
64. In trapezoid ABCD with bases AB and CD, we have $AB = 52$, $BC = 12$, $CD = 39$, and $DA = 5$. The area of ABCD is given by $10A$. Find the value of A .



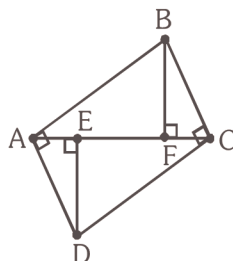
65. A regular octagon ABCDEFGH has sides of length two. Then the area of $\triangle ADG$ is given by $(a + b\sqrt{c})$. Find the product of a , b & c .
66. A regular octagon is formed by cutting an isosceles right triangle from each of the corners of a square with sides of length 20. Let the length of each side of the octagon is given by $a(\sqrt{b} - c)$. Find the product of a , b and c .
67. A right circular cylinder with its diameter equal to its height is inscribed in a right circular cone. The cone has diameter 10 and altitude 12, and the axes of the cylinder and cone coincide. Then the radius of the cylinder is R . Find the value of $(11R)$.
68. In trapezoid ABCD, \overline{AB} and \overline{CD} are perpendicular to \overline{AD} , with $AB + CD = BC$, $AB < CD$, and $AD = 7$. What is $(4AB \cdot CD)$?
69. A square is cut into three rectangles along two lines parallel to a side, as shown. If the perimeter of each of the three rectangles is 24, then the area of the original square is



70. In the figure, ABCD is an isosceles trapezoid with side lengths $AD = BC = 5$, $AB = 4$, and $DC = 10$. The point C is on \overline{DF} and B is the midpoint of hypotenuse \overline{DE} in the right triangle DEF. Then $CF =$

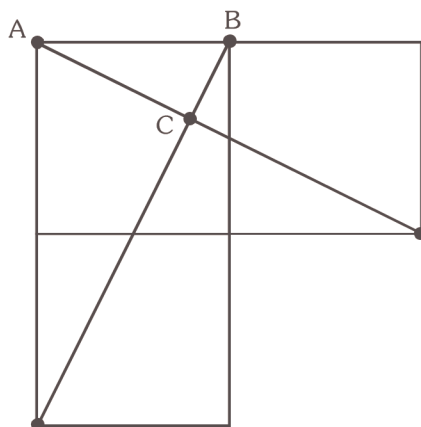


71. In the figure, ABCD is a quadrilateral with right angles at A and C. Points E and F are on \overline{AC} , and \overline{DE} and \overline{BF} are perpendicular to \overline{AC} . If $AE = 3$, $DE = 5$ and $CE = 7$, then the value of $BF - \frac{1}{5}$ is

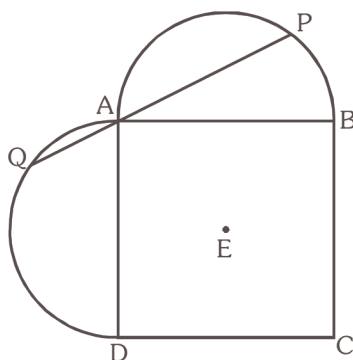


92. A wire is cut into two pieces, one of length a and the other of length b . The piece of length a is bent to form an equilateral triangle, and the piece of length b is bent to form a regular hexagon. The triangle and the hexagon have equal area. What is $\frac{4a^2}{b^2}$?

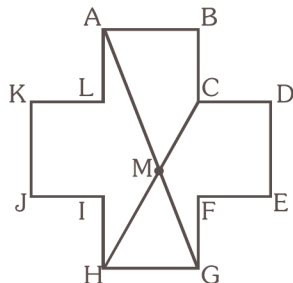
93. Three unit squares and two line segments connecting two pairs of vertices are shown. If the area of $\triangle ABC$ is N , find $5N$.



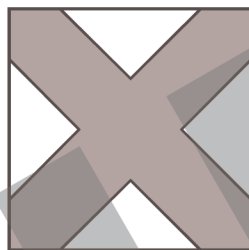
94. In rectangle $ABCD$, $AB = 6$, $AD = 30$, and G is the midpoint of \overline{AD} . Segment AB is extended 2 units beyond B to point E , and F is the intersection of \overline{ED} and \overline{BC} . If the area of $BFDG$ is X , find $[X]$.
95. Square $EFGH$ has one vertex on each side of square $ABCD$. Point E is on \overline{AB} with $AE = 7$. EB . If the ratio of the area of $EFGH$ to the area of $ABCD$ is $\frac{m}{n}$, where m, n are co-prime, find $m + n$.
96. Rectangle $ABCD$ has $AB = 6$ and $BC = 3$. Point M is chosen on side AB so that $\angle AMD = \angle CMD$. What is the degree measure of $\angle AMD$?
97. Points A and C lie on a circle centered at O , each of \overline{BA} and \overline{BC} are tangent to the circle, and $\triangle ABC$ is equilateral. The circle intersects \overline{BO} at D . What is $BD + BO$?
98. Suresh has 30 thin rods, one each of every integer length from 1 cm through 30 cm. She places the rods with lengths 3 cm, 7 cm, and 15 cm on a table. She then wants to choose a fourth rod that she can put with these three to form a quadrilateral with positive area. How many of the remaining rods can she choose as the fourth rod?
99. Equiangular hexagon $ABCDEF$ has side lengths $AB = CD = EF = 1$ and $BC = DE = FA = r$. The area of $\triangle ACE$ is 70% of the area of the hexagon. What is the sum of all possible values of r ?
100. In the following diagram, $ABCD$ is a square, and E is the center of the square $ABCD$. P is a point on a semi-circle with diameter AB . Q is a point on a semi-circle with diameter AD . Moreover, Q, A and P are collinear (that is, they are on the same line). Suppose $QA = 14$ cm, $AP = 46$ cm, and $AE = x$ cm. Find the value of x .



- 118.** Convex quadrilateral ABCD has $AB = 9$ and $CD = 12$. Diagonals AC and BD intersect at E, $AC = 14$, and $\triangle AED$ and $\triangle BEC$ have equal areas. What is AE?
- 119.** Consider the 12-sided polygon ABCDEFGHIJKL, as shown. Each of its sides has length 4, and each two consecutive sides form a right angle. Suppose that \overline{AG} and \overline{CH} meet at M. What is the area of quadrilateral ABCM to the nearest integer?



- 120.** A paint brush is swept along both diagonals of a square to produce the symmetric painted area, as shown. Half the area of the square is painted. If the ratio of the side length of the square to the brush width is $m\sqrt{2} + n$ then find the value of $(m^2n)^2$?



ANSWER KEY

EXERCISE - 4.1

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	3	4	2	1	2	4	3	1	2	1	1	3	1	1

15.8	16.0	17.70	19.30	20. $AC = 3\sqrt{5}$, $AB = \sqrt{13}$, $BC = 2\sqrt{13}$		
21. 4 : 1	26.20	27.48	28.54	29.33	30.15	31.01
32.12	33.87	34.03	35.12	36.26	37.30	38.20
39.01	40.03	41.06	42.35	43.20	44.02	45.50
46.15	47.02	48.70	49.77	50.52	51.12	52.03
53.20	54.18	55.14	56.02	57.75	58.12	59.80
60.37	61.05	62.48	63.05	64.14	65.56	66.34
67.36	68.21	69.15	70.30	71.10	72.48	73.00
74.08	75.12	76.04	77.27	78.02	79.33	80.05
81.01	82.24	83.21	84.27	85.90	86.02	87.03
88.82	89.03	90.15	91.01	92.12	93.56	94.24
95.48	96.02	97.43	98.18	99.03	100.23	101.04
102.65	103.27	104.12	105.05	106.09	107.26	108.30
109.64	110.07	111.10	112.05	113.03	114.36	115.40
116.15	117.12	118.10	119.60	120.15	121.13	122.20
123.75	124.50	125.72	126.54	127.37	128.33	129.75

EXERCISE - 4.2

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	2	2	1	2	1	1	4	1	3	4	4	2	1	4	3
Que.	16	17	18	19	20	21	22	23	24						
Ans.	1	2	3	1	1	2	2	1	2						

25.20	26.57	27.36	28.60	29.04	30.05	31.07
32.49	33.17	34.03	35.53	36.72	37.66	38.40
39.36	40.61	41.30	42.63	43.08	44.16	45.54
46.65	47.60	48.78	49.45	50.8	51.35	52.82
53.10	54.11	55.04	56.02	57.23	58.44	59.24
60.02	61.60	62.04	63.05	64.03	65.06	66.92
67.01	68.73	69.02	70.06	71.27	72.16	73.03
74.69	75.19	76.08	77.02	78.08	79.12	80.03
81.02	82.26	83.06	84.06			

EXERCISE - 4.3

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ans.	1	1	3	1	4	3	3	2	2	1	2	1	1	1

15. $\frac{12}{5}$	16.54	17.60	18.25	19.40	20.12	
21.16	22.19	23.01	24.02	25.25	26.30	27.31
28.16	29.21	30.07	31.03	32.50	33.16	34.15
35.11	36.20	37.31	38.24	39.42	40.41	41.28
42.26	43.24	44.96	45.23	46.10	47.03	48.09
49.04	50.19	51.98	52.85	53.86	54.06	55.01
56.04	57.08	58.75	59.05	60.02	61.05	62.20
63.25	64.21	65.24	66.40	67.30	68.49	69.81
70.04	71.04	72.07	73.01	74.34	75.62	76.09
77.22	78.98	79.45	80.20	81.50	82.64	83.81
84.48	85.31	86.10	87.72	88.06	89.05	90.25
91.49	92.6	93.01	94.67	95.57	96.75	97.03
98.17	99.6	100.34	101.45	102.84	103.06	104.05
105.12	106.08	107.01	108.80	109.04	110.25	111.24
112.02	113.26	114.10	115.27	116.27	117.10	118.06
119.18	120.64					